Exponential Growth and Decay Models

***Law of Unhibited Growth or Decay*:**
Many natural phenomena have been found to follow the law that an amount A varies with time t according to the function $A(t) = A_0 e^{kt}$, where A_0 is the original amount at time $t = 0$ and k is a constant of growth or decay (growth if $k > 0$, decay if $k < 0$.)

Example: The number N of bacteria present in a culture at time t hours obeys the law of uninhibited growth where $N(t) = 1000e^{0.01t}$.

a) Determine the number of bacteria at $t = 0$ hours.

b) What is the growth rate of the bacteria?

c) What will the population be after 4 hours?

d) When will the number of bacteria reach 1700?

e) When will the number of bacteria double?

Example: The annual growth rate of the world's population in 2005 was $k = 1.15\% = 0.0115$. The population of the world in 2005 was 6,451,058,790 people. Letting $t = 0$ represent the year 2005, use the uninhibited growth model to predict the world's population in the year 2015.

Example: Iodine 131 is a radioactive material that decays according to the function $A(t)=A_0e^{-0.087t}$, where A_0 is the initial amount present and A is the amount present at time t (in days). Assume that a scientist has a sample of 100 grams of iodine 131.

a) What is the decay rate of iodine 131?

b) How much iodine 131 is left after 9 days?

c) When will 70 grams of iodine 131 be left?

d) What is the half-life of iodine 131? (when $A=\frac{1}{2}A_0$.)

Example: A piece of charcoal contains 30% of the carbon 14 that it originally had. When did the tree die from which the charcoal came? Use 5600 years as the half-life of carbon 14.

Example: At 45°C, dinitrogen pentoxide decomposes into nitrous dioxide and oxygen according to the law of uninhibited decay. An initial amount of 0.25 M of dinitrogen pentoxide decomposes to 0.15 M in 17 minutes. How much dinitrogen pentoxide will remain after 30 minutes? How long will it take until 0.01 M of dinitrogen pentoxide remains?

***Logistic Models*:**
In many cases, growth or decay does not occur without limit. For example, cell division is often limited by factors like living space and food supply. In these cases, we use ***logistic models,*** which can describe situations where the growth or decay of the dependent variable is limited.

The population P after time t is given by $P(t)=\dfrac{c}{1+ae^{-bt}}$, where a, b, and c are constants with $a>0$ and $c>0.$

- c is called the ***carrying capacity*.** It is the upper limit of the population. In a growth model, the population approaches c as $t\to\infty.$
- $|b|$ is the growth or decay rate, as a decimal. If $b>0,$ the model is a growth model. If $b<0,$ the model is a decay model.

Example: The logistic growth model $P(t)=\dfrac{1000}{1+32.33e^{-0.439t}}$ represents the population (in grams) of a bacterium after t hours.

a) Determine the carrying capacity of the environment.

b) What is the growth rate of the bacteria?

c) Determine the initial population size.

d) What is the population after 9 hours?

e) When will the population be 700 grams?

f) How long does it take for the population to reach one-half the carrying capacity?